MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 2, 2019/2020 SESSION

PTG 0116 – TRIGONOMETRY AND COORDINATE GEOMETRY
(All sections / Groups)

05 MARCH 2020
2:30p.m. – 4:30 p.m.
(2 Hours)

INSTRUCTIONS TO STUDENT

1. This question paper consists of FOUR pages with FOUR questions and an appendix only.

2. Answer ALL questions.

3. Unless stated otherwise, if an answer is given as a decimal, it should be rounded to **four** significant figures.

4. Write your answers in the Answer Booklet provided.

5. Show all relevant steps to obtain maximum marks.

Question 1

(a) Obtain the exact value of $\tan(\cos^{-1}\frac{\sqrt{3}}{2})$ [4 marks]

(b) Determine the amplitude and period of the following function

$$y = 3\cos(2x + \pi)$$

Sketch the graph of the function and label all the key points for at the least one cycle.

                                                             [8 marks]

(c) Show that $\dfrac{\sin 3\alpha - \sin \alpha}{\cos 3\alpha - \cos\alpha} = -\cot 2\alpha$ [6 marks]

(d) A ball bounces off the table as shown in **Figure 1.**

Given that $\theta_1 = 30°$, $L_1 = 20$ cm and $L_3 = 200$ cm, determine θ_2, θ_3 and L_2.

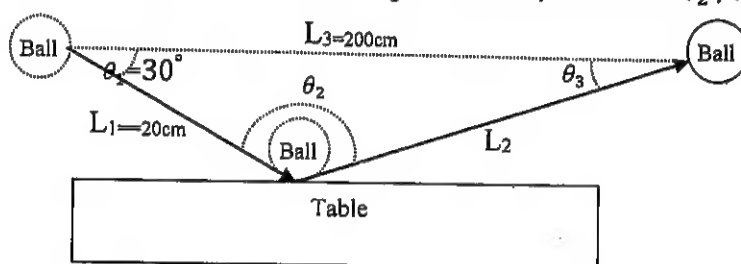**Figure 1**

                                                   [7 marks]

Question 2

(a) Express polar coordinates $(5, \frac{2\pi}{3})$ in Cartesian coordinates system.   [3 marks]

(b) Express the complex number $\dfrac{2+3i}{2-3i}$ in polar form.   [6 marks]

(c) Find the roots of the following equation:

$$x^2 + 8x + 25 = 0$$ [5 marks]

(d) If $p = 3i + 5j - 7k$ and $q = 2i - 6j + 4k$. Find

  (i) $p \cdot q$

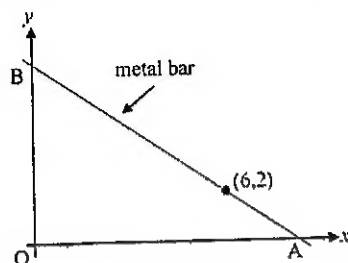 (ii) $p \times q$ [3 marks]
                                                       [5 marks]

  (iii) $p \cdot r$ if $r = p \times q$ [3 marks]

Continued...

Question 3

(a) A metal bar 9 cm long places with one end A on the x-axis and the other end B on the y-axis. It passes through the point (6,2). If O is the origin O (0,0) and OA is x, derive the distance OB in terms of x as shown in **Figure 3**.

Figure 3

Hence show that the length of the metal bar can be expressed as

$$x^2 + \frac{4x^2}{(x-6)^2} = 81$$ [9 marks]

(b) Obtain the centre and radius of the circle $x^2 + y^2 + 8x - 4y = -16$

Then, determine whether the point (–4, -2) is inside, on, or outside the circle.
                                                        [8 marks]

(c) Given an Ellipse equation,

$$\frac{x^2}{9} + \frac{y^2}{16} = 1$$

Calculate
  (i) the eccentricity
  (ii) the coordinates of the foci
  (iii) the equations of the directrices   [8 marks]

Question 4

(a) By the use of Cramer's rule, solve the following equations

$$3x - y + 4z = -2$$
$$x + 2y - z = -3$$
$$-2x + 3y + z = 5$$

                                                        [13 marks]

(b) The determinants B is given as=

$$B = \begin{bmatrix} 4 & 0 & 5 \\ 8 & 2 & 6 \\ -12 & 9 & -11 \end{bmatrix}$$

Calculate
(i)   det (B),   [2 marks]
(ii)   the cofactors of each element in B,   [9 marks]
(iii) B^{-1}.   [1 mark]

                                                        Continued...

APPENDIX

$$\sin^2\theta + \cos^2\theta = 1$$
$$\tan^2\theta + 1 = \sec^2\theta$$
$$1 + \cot^2\theta = \csc^2\theta$$

$$\sin^2 A = \frac{1 - \cos 2A}{2}$$
$$\cos^2 A = \frac{1 + \cos 2A}{2}$$
$$\tan^2 A = \frac{1 - \cos 2A}{1 + \cos 2A}$$
$$\tan A = \frac{\sin 2A}{1 + \cos 2A}$$
$$= \frac{1 - \cos 2A}{\sin 2A}$$

$$\cos(A+B) = \cos A \cos B - \sin A \sin B$$
$$\cos(A-B) = \cos A \cos B + \sin A \sin B$$
$$\sin(A+B) = \sin A \cos B + \cos A \sin B$$
$$\sin(A-B) = \sin A \cos B - \cos A \sin B$$
$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$
$$\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

$$\sin A \cos B = \frac{1}{2}\left[\sin(A-B) + \sin(A+B)\right]$$
$$\cos A \cos B = \frac{1}{2}\left[\cos(A-B) + \cos(A+B)\right]$$
$$\sin A \sin B = \frac{1}{2}\left[\cos(A-B) - \cos(A+B)\right]$$

$$\sin 2A = 2 \sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A$$
$$= 2\cos^2 A - 1$$
$$= 1 - 2\sin^2 A$$
$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

$$\sin A + \sin B = 2 \sin\frac{A+B}{2}\cos\frac{A-B}{2}$$
$$\sin A - \sin B = 2 \cos\frac{A+B}{2}\sin\frac{A-B}{2}$$
$$\cos A + \cos B = 2 \cos\frac{A+B}{2}\cos\frac{A-B}{2}$$
$$\cos A - \cos B = -2 \sin\frac{A+B}{2}\sin\frac{A-B}{2}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\cos\alpha = \frac{b^2 + c^2 - a^2}{2bc}$$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\bar{p} \cdot \bar{q} = |\bar{p}||\bar{q}|\sin\theta$$

$$\bar{p} \times \bar{q} = |\bar{p}||\bar{q}|\cos\theta$$

General form of parabola:
$$y^2 = 4px \text{ or } x^2 = 4py \text{ , } p \text{ is the focus}$$

General form of circle:
$$(x-a)^2 + (y-b)^2 = r^2$$

General form of Hyperbola:

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \text{ transverse axis along x- axis foci } (\pm c, 0)$$
$$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1 \text{ transverse axis along y- axis Foci}(0, \pm c)$$

General form of Ellipse:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \text{ for a>b, Foci}(\pm c, 0) \text{ major axis: } x$$

$$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1 \text{ for a>b, Foci}(0, \pm c) \text{ major axis: } y$$

$$b^2 = c^2 - a^2 = a^2(e^2 - 1)$$

$$c^2 = a^2 - b^2 \text{ , Eccentricity= } e = \frac{c}{a}$$

Eccentricity= $e = \dfrac{c}{a}$

$$e^2 = \frac{a^2 + b^2}{a^2}$$

$$e^2 = \frac{a^2 + b^2}{a^2}$$

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